Exponents can be confusing at times, but once you understand the pattern, they become easier to work with. Let’s try a few examples:

Simplify the following expression: \( \frac{(2a^2bc^4)^2}{(2a^{-1}b^2c)^3} \)

The first thing we need to do is move the power from outside of the parentheses to the inside of the parentheses. To do this, we multiply the powers together (Remember: if there isn’t a power, then it’s safe to assume that there is an invisible one there). Let’s start by distributing the exponent on the numerator (the top part of the fraction) first:

\[
(2a^2bc^4)^2 = 2^{(1\times2)}a^{(2\times2)}b^{(1\times2)}c^{(4\times2)} = 2^2a^4b^2c^8
\]

Now let’s distribute the exponent in the denominator (the bottom part of the fraction):

\[
(2a^{-1}b^2c)^3 = 2^{(1\times3)}a^{(-1\times3)}b^{(2\times3)}c^{(1\times3)} = 2^3a^{-3}b^6c^3
\]

Okay, now we need to reconstruct our fraction:

\[
\frac{2^2a^4b^2c^8}{2^3a^{-3}b^6c^3}
\]

Now we need to reduce the fraction by comparing each value in our fraction (i.e. comparing the numbers, the a’s, the b’s, and the c’s). For this, we want to compare the exponents of each individual value. We want to subtract the exponent value from the exponent value on the top:

- \( 2: 2 - 3 = -1 \)
• $a: 4 - (-3) = 7$
• $b: 2 - 6 = -4$
• $c: 8 - 3 = 5$

The value we got is now the new value of the exponent for each value. For now, we want to put each part on top of the fraction (that is, we do not want a denominator at the moment):

$$= 2^{-1}a^7b^{-4}c^5$$

Now all we need to do is take the ones with negative exponents (in our example: 2 and b) change their sign from negative to positive, and put them on the bottom of the fraction (note: since the power 2 is 1, we can leave it out, since $2^1 = 2$):

$$= \frac{a^7c^5}{2b^4}$$

And now our expression is reduced. A more generalized form of these rules are as follows:

- $(x^A)^B = x^{AB}$
- $x^A x^B = x^{A+B}$
- $\frac{x^A}{x^B} = x^{A-B}$
- $(AB)^x = A^x B^x$

For a video on this, please reference [https://www.youtube.com/watch?v=kITJ6qH7jS0](https://www.youtube.com/watch?v=kITJ6qH7jS0)
SOLVING LOGARITHMS AND NATURAL LOGS

Logarithms may seem hard to use, but they in fact make it very easy for us to work with larger numbers. Let’s look at a few examples on how to solve logarithms and natural logs:

Determine the value of $x$ in the following equation: $\log_{x} 100 = 2$.

The first thing we must do is rewrite the equation. We can do this by taking the base (in our example: $x$) and raising it to the right-hand side (in our example: 2), and setting it equal to the value that we are taking the log of (in our example: 100).

$$x^2 = 100$$

Now we can solve for $x$:

$$x^2 = 100, \quad x = 10$$

Therefore, $\log_{10} 100 = 2$. (Note, some teachers will write $\log_{10} a$ as $\log a$, where $a$ is an arbitrary value.)

Let’s try another example, but this time we will attempt to find a different unknown:

Determine the value of $x$ where $\ln x = 1$.

We should start by noting that $\ln x$ is the same thing as $\log_e x$. Now all we need to do is rewrite our equation the same way we did in the previous example:

$$e^1 = x$$

Now we can solve for $x$: 
\[e^1 = x, \quad e = x\]

Therefore, \(\ln e = 1\).

A general form of this is \(\log_A B = C, \text{ then } A^C = B\).

For a video on this, please reference https://www.youtube.com/watch?v=mQTWzLpCcW0
USING OPERATIONS WITH LOGARITHMS

Using operations with logarithms may seem unnatural and strange, but remember, logarithms are a way to find and solve for exponents. Let’s look at a few examples:

Simplify the expression \( \log_a c + \log_a b + \log_c b - \log_c a \).

The first thing we need to do is identify the logarithmic expressions that have the same base. We are only allowed to combine logarithmic expressions only if they have the same base:

- Base \( a \): \( \log_a c, \log_a b \)
- Base \( c \): \( \log_c b, \log_c a \)

Now, logarithms have a very special property about them. If two logarithmic expressions have the same base and we are trying to add them together, then we can multiply the values that we are taking the logarithm of. If they have the same base and we are trying to subtract them, then we can divide the values that we are trying to take the logarithm of:

- \( \log_a c + \log_a b = \log_a (bc) \)
- \( \log_c b - \log_c a = \log_c \left( \frac{b}{a} \right) \)

So, for our expression:

\[
\log_a c + \log_a b + \log_c b - \log_c a = \log_a (bc) + \log_c \left( \frac{b}{a} \right)
\]

Let’s try another example:

Solve for \( x \): \( \log_x 8^2 = 6 \)
A special property about logarithms is that we can “drop” the exponent (that is, we can move the exponent in front of the logarithm, making it a coefficient):

\[ 2 \log_x 8 = 6 \]

We can remove the coefficient by dividing through by two on both sides:

\[ \log_x 8 = 3 \]

Now we want to rewrite our expression:

\[ x^3 = 8 \]

Now we solve for \( x \):

\[ x^3 = 8, \quad x = 2 \]

Therefore, \( \log_x 8^2 = 6 \).

For a video on this, please reference [https://www.youtube.com/watch?v=RhzXX5PbsuQ](https://www.youtube.com/watch?v=RhzXX5PbsuQ)
CHANGE OF BASE FOR LOGARITHMS

Using logarithms can be difficult sometimes, but sometimes if we change the base of our logarithm it makes things simpler. Let’s look at an example:

Determine what \( \log_3 100 \) is using the change of base formula.

First off we need to identify the change of base formula. The change of base formula states that

\[
\frac{\log_b a}{\log_a b} = \log_a b
\]

Where \( x \) is an arbitrary number. Since logarithms are typically simpler when done in base 10, let’s change our expression into base 10.

\[
\log_3 100 = \frac{\log_{10} 100}{\log_{10} 3}
\]

We know from earlier studies with logarithms that \( \log_{10} 100 = 2 \). Finding \( \log_{10} 3 \) by hand is quite a cumbersome task, so we will use our calculator to determine that \( \log_{10} 3 = 0.477 \).

(Note: Most calculators in the United States will denote \( \log_{10} x \) as just \( \log(x) \). That is, if you put \( \log(3) \) in your calculator, you are actually calculating \( \log_{10}(x) \) .) We can now replace our new values and solve our expression:

\[
\log_3 100 = \frac{2}{0.477}
\]

\[
\log_3 100 \approx 4.19
\]

For a video on this, please reference https://www.youtube.com/watch?v=OkFdDqW9xxM